

A generalized growth index parametrization: Applications to the Λ CDM using the *WiggleZ* growth data

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We re-examine the growth index of the concordance Λ cosmology in the light of the latest *WiggleZ* data. In particular, we investigate five different models for the growth index γ , by comparing their cosmological evolution using observational data of the growth rate of structure formation at different redshifts, consistently scaled to the WMAP7 cosmology. Using a χ^2 minimization procedure between the different growth rate parametrizations and the data we determine the free parameters of the models and we statistically quantify their ability to represent the observations. Using the combined growth data provided by the 2dF, SDSS and *WiggleZ* galaxy surveys and assuming a constant growth index we find that $\gamma = 0.616^{+0.088}_{-0.083}$. Based on *WiggleZ* data alone we obtain $\gamma = 0.64^{+0.099}_{-0.094}$, which is somewhat greater, and almost 1σ away, of the theoretically predicted value of $\gamma \simeq 6/11$. Under the assumption that the growth index varies with time we find that the addition of the *WiggleZ* growth data in the likelihood analysis improves significantly the statistical results. As an example, in the case of $\gamma(z) = \gamma_0 + \gamma_1 z$ we find $\gamma_0 = 0.684^{+0.136}_{-0.124}$ and $\gamma_1 = -0.331^{+0.507}_{-0.448}$, while without the *WiggleZ* we find that the parameters (γ_0, γ_1) remain unconstrained.

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1. INTRODUCTION

The high-quality cosmological observational data (e.g. supernovae type Ia, CMB, galaxy clustering, etc), accumulated during the last two decades, have enabled cosmologists to gain substantial confidence that modern cosmology is capable of quantitatively reproducing the details of many observed cosmic phenomena, including the late time accelerating stage of the Universe. A variety of studies have converged to a cosmic expansion history involving a spatially flat geometry and a cosmic dark sector formed by cold dark matter and some sort of dark energy, endowed with large negative pressure, in order to explain the observed accelerating expansion of the Universe [1–9] and references therein).

In spite of that, the absence of a fundamental physical theory, regarding the mechanism inducing the cosmic acceleration, has given rise to a plethora of alternative cosmological scenarios. Most are based either on the existence of new fields in nature (dark energy) or in some modification of Einstein's general relativity, with the present accelerating stage appearing as a sort of geometric effect. In order to test the latter possibilities, it has been proposed that measuring the so called growth index, γ , could provide an efficient way to discriminate between modified gravity models and dark energy (hereafter DE) models which adhere to general relativity. The accurate determination of the growth index is considered one of the most fundamental tasks on the interface between Astronomy and Cosmology. Its importance stems from the fact that there is only a weak dependence of γ on the equation of state parameter $w(z)$, as has been found in Linder & Cahn [10], which implies

that one can separate the background expansion history, $H(z)$, constrained by a large body of cosmological data (SNIa, BAO, CMB), from the fluctuation growth history, given by γ . As an example, it was theoretically shown that for DE models within general relativity the growth index γ is well approximated by $\gamma \simeq \frac{3(w-1)}{6w-5}$ (see [11],[10],[12]), which boils down to $\approx 6/11$ for the Λ CDM cosmology $w(z) = -1$. Notice, that in the case of the braneworld model of Dvali, Gabadadze & Porrati [13] we have $\gamma \approx 11/16$ (see also [10, 14]), while for the $f(R)$ gravity models we have $\gamma \simeq 0.41 - 0.21z$ for $\Omega_{m0} = 0.27$ [15].

From the observational viewpoint, indirect methods to measure γ have also been developed (mostly using a constant γ), based either on the observed growth rate of clustering [12, 16–19] providing a wide range of γ values $\gamma = (0.6 - 0.67)^{+0.40}_{-0.30} {}^{+0.20}_{-0.17}$, or on massive galaxy clusters Vikhlinin et al. [20] and Rapetti et al. [21] with the latter study providing $\gamma = 0.42^{+0.20}_{-0.16}$, or even on the weak gravitational lensing [22]. Gaztanaga et al., [23] performed a cross-correlation analysis between probes of weak gravitational lensing and redshift space distortions and found no evidence for deviations from general relativity. With the next generation of surveys, based on *Euclid* and *BigBOSS*, we will be able to put strong constraints on γ (see for example [24–26] and references therein) and thus to test the validity of general relativity on cosmological scales.

In this article, we wish to test some basic functional forms of $\gamma(z)$ in the light of the *WiggleZ* growth rate data. The structure of the paper is as follows. Initially in section 2, we briefly discuss the background cosmological equations. The basic theoretical elements of the growth index are presented in section 3, where we extend the original Polarski & Gannouji method [27] for a general family of $\gamma(z)$ parametrizations. In section 4, a joint statistical analysis based on the observed linear

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growth rate of clustering, measured from the 2dF, SDSS and *WiggleZ* redshifts catalogs, is used to constraint the growth index model free parameters. Finally, we draw our main conclusions in section 5.

2. BASIC COSMOLOGICAL EQUATIONS

In this section, it will be assumed that the universe is a self-gravitating fluid described by general relativity, and endowed with a spatially flat homogeneous and isotropic geometry. In addition, we also consider that it is filled by non-relativistic matter plus a DE component (or some effective mechanism that simulates it), and whose equation of state (hereafter EoS), $p_{DE} = w(a)\rho_{DE}$, is driving the present accelerating stage. Following standard lines, the Hubble flow reads:

$$\frac{H^2(a)}{H_0^2} \equiv E^2(a) = \Omega_{m0}a^{-3} + \Omega_{DE0}e^{3\int_a^1 \frac{d\ln y}{y}[1+w(y)]}, \quad (2.1)$$

where $a(z) = 1/(1+z)$ is the scale factor of the universe, $E(a)$ is the normalized Hubble flow, Ω_{m0} is the dimensionless matter density at the present epoch, $\Omega_{DE0} = 1 - \Omega_{m0}$ denotes the DE density parameter, and $w(a)$ its EoS parameter. On the other hand, we can express the EoS parameter in terms of $E(a) = H(a)/H_0$ [28] using the Friedmann equations as

$$w(a) = \frac{-1 - \frac{2}{3}a \frac{d\ln E}{da}}{1 - \Omega_m(a)} \quad (2.2)$$

where

$$\Omega_m(a) = \frac{\Omega_{m0}a^{-3}}{E^2(a)}. \quad (2.3)$$

Differentiating the latter and utilizing Eq. (2.2) we find that

$$\frac{d\Omega_m}{da} = \frac{3}{a}w(a)\Omega_m(a)[1 - \Omega_m(a)]. \quad (2.4)$$

Since the exact nature of the DE has yet to be found, the above DE EoS parameter encodes our ignorance regarding the physical mechanism powering the late time cosmic acceleration.

The methodology described above can also be applied to the framework of modified gravity (see [29, 30]). In this case, instead of using the exact Hubble flow through a modification of the Friedmann equation one may consider an equivalent Hubble flow somewhat mimicking Eq. (2.1). The key point here is that the accelerating expansion can be attributed to a kind of “geometrical” DE contribution. Now, since the matter density (baryonic+dark) cannot accelerate the cosmic expansion, we perform the following parametrization [29, 30]:

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_{m0}a^{-3} + \Delta H^2. \quad (2.5)$$

Naturally, any modification to the Friedmann equation of general relativity may be included in the last term of the above expression. After some algebra one may also derive, using Eqs. (2.2) and (2.5), an effective (“geometrical”) dark energy EoS parameter, given by:

$$w(a) = -1 - \frac{1}{3} \frac{d\ln \Delta H^2}{d\ln a}. \quad (2.6)$$

Notice that we will use the above quantities in the next section.

3. THE EVOLUTION OF THE LINEAR GROWTH FACTOR

Here, we briefly discuss the basic equation which governs the behavior of the matter perturbations on sub-horizon scales and within the framework of any DE model, including those of modified gravity (“geometrical dark energy”). At the sub-horizon scales the DE component is expected to be smooth and thus it is fair to consider perturbations only on the matter component of the cosmic fluid [31]. Consequently, the evolution equation of the matter fluctuations, for models where the DE fluid has a vanishing anisotropic stress and the matter fluid is not coupled to other matter species (see [32],[33],[34],[35],[10],[36],[15]), is given by:

$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{\text{eff}}\rho_m\delta_m \quad (3.1)$$

where ρ_m is the matter density and $G_{\text{eff}}(t) = G_N Q(t)$, with G_N denoting Newton’s gravitational constant.

For those cosmological models which adhere to general relativity, [$Q(t) = 1$, $G_{\text{eff}} = G_N$], the above equation reduces to the usual time evolution equation for the mass density contrast [37], while in the case of modified gravity models (see [32],[10], [36],[15]), we have $G_{\text{eff}} \neq G_N$ (or $Q(t) \neq 1$). In this context, $\delta_m(t) \propto D(t)$, where $D(t)$ is the linear growing mode (usually scaled to unity at the present time). If we change the variables from t to a ($\frac{d}{dt} = H \frac{d}{d\ln a}$) then the time evolution of the mass density contrast (see Eq. (3.1)) takes the following form

$$\frac{a^2}{\delta_m} \frac{d^2 \delta_m}{da^2} + \left(3 + a \frac{d\ln E}{da}\right) \frac{a}{\delta_m} \frac{d\delta_m}{da} = \frac{3}{2} \Omega_m(a) Q(a). \quad (3.2)$$

Solving Eq.(3.1 or 3.2) for the concordance Λ cosmology¹, we derive the well known perturbation growth factor (see [37]):

$$D(z) = \frac{5\Omega_{m0}E(z)}{2} \int_z^{+\infty} \frac{(1+u)du}{E^3(u)}. \quad (3.3)$$

¹ For the usual Λ CDM cosmological model we have $w(a) = -1$, $\Omega_\Lambda(a) = 1 - \Omega_m(a)$ and $Q(a) = 1$.

Obviously, for $E(z) \simeq \Omega_{m0}^{1/2} (1+z)^{3/2}$ it gives the standard result $D(z) \simeq a = (1+z)^{-1}$, which corresponds to the matter dominated epoch, as expected.

Now, for any type of DE, an efficient parametrization of the matter perturbations is based on the growth rate of clustering [37]

$$f(a) = \frac{d \ln \delta_m}{d \ln a} \simeq \Omega_m^\gamma(a) \quad (3.4)$$

where γ is the so called growth index (see Refs. [10–12, 29, 32]) which plays a key role in cosmological studies as we described in the introduction, especially in the light of recent large redshift surveys (like the *WiggleZ*; see [38, 39] and references therein).

A. The generalized growth index parametrization

Inserting the first equality of Eq.(3.4) into Eq. (3.2) and using simultaneously Eq. (2.2), we arrive after some algebra, at

$$a \frac{df}{da} + f^2 + X(a)f + f^2 = \frac{3}{2} \Omega_m(a) Q(a), \quad (3.5)$$

where

$$X(a) = \frac{1}{2} - \frac{3}{2} w(a) [1 - \Omega_m(a)]. \quad (3.6)$$

Now, we consider that the growth index varies with cosmic time. Transforming equation (3.5) from a to redshift $[\frac{d}{da} = -(1+z)^{-2} \frac{d}{dz}]$ and utilizing Eqs.(3.4) (2.4), we simply derive the evolution equation of the growth index $\gamma = \gamma(z)$ (see also [27]). Indeed this is given by:

$$\begin{aligned} & -(1+z)\gamma' \ln(\Omega_m) + \Omega_m^\gamma + 3w(1 - \Omega_m)(\gamma - \frac{1}{2}) + \frac{1}{2} \\ & = \frac{3}{2} Q \Omega_m^{1-\gamma}, \end{aligned} \quad (3.7)$$

where prime denotes derivative with respect to redshift. At the present epoch the above equation takes the form:

$$\begin{aligned} & -\gamma'(0) \ln(\Omega_{m0}) + \Omega_{m0}^{\gamma(0)} + 3w_0(1 - \Omega_{m0})[\gamma(0) - \frac{1}{2}] + \frac{1}{2} \\ & = \frac{3}{2} Q_0 \Omega_{m0}^{1-\gamma(0)}, \end{aligned} \quad (3.8)$$

where $Q_0 = Q(z=0)$ and $w_0 = w(z=0)$.

Over, the last few years there have been many theoretical speculations regarding the functional form of the growth index and indeed various candidates have been proposed in the literature. Here we phenomenologically parametrize $\gamma(z)$ by the following general relation

$$\gamma(z) = \gamma_0 + \gamma_1 y(z). \quad (3.9)$$

The latter equation can be seen as a first order Taylor expansion around some cosmological quantity such as $a(z)$, z and $\Omega_m(z)$. Interestingly, for those $y(z)$ functions

which satisfy $y(0) = 0$ [or $\gamma(0) = \gamma_0$] one can write the parameter γ_1 in terms of γ_0 . In this case $[\gamma'(0) = \gamma_1 y'(0)]$, using Eq.(3.8) we obtain

$$\gamma_1 = \frac{\Omega_{m0}^{\gamma_0} + 3w_0(\gamma_0 - \frac{1}{2})(1 - \Omega_{m0}) - \frac{3}{2} Q_0 \Omega_{m0}^{1-\gamma_0} + \frac{1}{2}}{y'(0) \ln \Omega_{m0}}. \quad (3.10)$$

Note that for the rest of the paper we concentrate on the usual Λ CDM cosmology and thus we set $Q(z) = 1$.

Let us now briefly present various forms of $\gamma(z)$, $\forall z$.

- Expansion around $z = 0$ (see [27]; hereafter Γ_1 model): In this case we have $y(z) = z$. Note however, that this parametrization is valid at relatively low redshifts $0 \leq z \leq 0.5$. In the statistical analysis presented below we utilize a constant γ namely $\gamma = \gamma_0$ for $z > 0.5$.
- Interpolated parametrization (hereafter Γ_2 model): Since Γ_1 model is valid at low redshifts we propose to use a new formula $y(z) = ze^{-z}$ that connects smoothly low and high-redshifts ranges. The above formula can be viewed as a combination of Γ_1 model with that of Dossett et al.[19]. For $z \gg 1$ we have $\gamma_\infty \simeq \gamma_0$.
- Expansion around $a = 1$ ([25, 26]; hereafter Γ_3 model): Here the function y becomes $y(z) = 1 - a(z) = \frac{z}{1+z}$. Obviously, at large redshifts $z \gg 1$ we get $\gamma_\infty \simeq \gamma_0 + \gamma_1$.
- Expansion around $\Omega_m = 1$ ([11]; hereafter Γ_4 model): In this parametrization we have $y(z) = 1 - \Omega_m(z)$. For a constant equation of state parameter $w(z) = w_0$ one can write (γ_0, γ_1) in terms of w_0

$$\gamma_0 = \frac{3(1-w_0)}{5-6w_0} \quad \gamma_1 = \frac{3}{125} \frac{(1-w_0)(1-3w_0/2)}{(1-6w_0/5)^3}. \quad (3.11)$$

Since at large redshifts $\Omega_m \simeq 1$ we can write $\gamma_\infty \simeq \gamma_0$.

To conclude, for the Γ_1 , Γ_2 and Γ_3 parametrizations one can show that $y(0) = 0$ and $y'(0) = 1$, respectively. Therefore, for the case of the Λ CDM cosmology with $\gamma_0 \simeq 6/11$ and $\Omega_{m0} = 0.273$, Eq.(3.10) provides $\gamma_1 \simeq -0.0478$, while for the case of the Γ_4 model we obtain $\gamma_1 \simeq 0.01127$ (see Eq.3.11).

4. OBSERVATIONAL CONSTRAINTS

In the following we briefly present some details of the statistical method and on the observational sample that we adopt in order to constrain the free parameters of the growth index, presented in the previous section. In order to quantify the growth index we perform a standard χ^2 minimization procedure between the observationally

TABLE I: Data of the growth rate of clustering. The correspondence of the columns is as follows: number, redshift, observed growth rate, cosmological parameters used by different authors and references.

Index	z	$f_{obs,Ref}$	$(\Omega_{m,Ref}, \sigma_{8,Ref})$	Refs.
1	0.15	0.49 ± 0.14	(0.30, 0.90)	[16, 40, 41]
2	0.35	0.70 ± 0.18	(0.24, 0.76)	[42]
3	0.55	0.75 ± 0.18	(0.30, 1.00)	[43]
4	0.77	0.91 ± 0.36	(0.27, 0.78)	[16]
5	1.40	0.90 ± 0.24	(0.25, 0.84)	[44]
6	2.42	0.74 ± 0.24	(0.26, 0.93)	[19, 45]
7	3.00	1.46 ± 0.29	(0.30, 0.85)	[46]
8	0.22	0.60 ± 0.10	(0.27, 0.80)	[38]
9	0.41	0.70 ± 0.07	(0.27, 0.80)	[38]
10	0.60	0.73 ± 0.07	(0.27, 0.80)	[38]
11	0.78	0.70 ± 0.08	(0.27, 0.80)	[38]

measured growth rate (based on the 2dF, SDSS and *WiggleZ* redshift surveys) and that expected in the Λ CDM cosmological model, according to:

$$\chi^2(z_i|\mathbf{p}) = \sum_{i=1}^N \left[\frac{f_{obs}(z_i) - \Omega_m(z_i)^{\gamma(z_i, \mathbf{p})}}{\sigma_i} \right]^2, \quad (4.1)$$

where N is the number of entries used in the statistical analysis² and σ_i is the observed growth rate uncertainty.

Evidently, the essential free parameters that enter in the theoretical expectation of Eq.(3.4) are: $(\Omega_{m0}, \gamma_0, \gamma_1) \equiv \mathbf{p}$. In Table I we quote the precise numerical values of the data points with the corresponding errors bars. This is an expanded version of the data-set used in [12, 18, 47] in which we have included data from the *WiggleZ* [38] galaxy survey (see entries 8–11 in Table I). The former sample measures $f(z)$ to within 20–40% while the latter 9–17%. The observed growth rate of structure ($f_{obs} = \beta b$) is derived from the redshift space distortion parameter $\beta(z)$ and the linear bias $b(z)$. The distortion is measured by the redshift-space two-point correlation function while the bias factor is estimated by comparing either the observed mass-tracer and a reference underlying mass power-spectrum of fluctuations (usually the Λ CDM model) or its Fourier transform, the correlation function [48].

In Appendix A we present a rescaling method used in order to transform different growth rate data to the same (WMAP7) cosmology (ie., flat Λ CDM with $\Omega_{m0} = 0.273$ and $\sigma_8 = 0.811$ [9]).

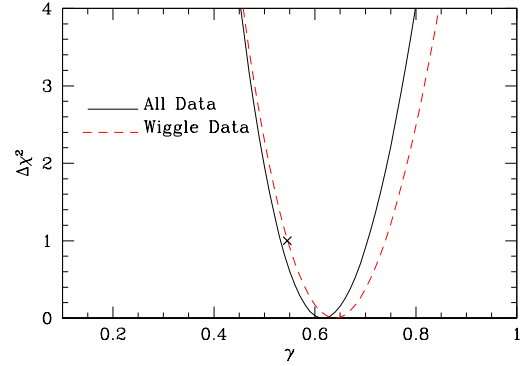


FIG. 1: The variance $\Delta\chi^2 = \chi^2 - \chi_{min}^2$ around the best fit γ value. The solid and the dashed line correspond to the total sample (see Table I) and to the *WiggleZ* data respectively. Note that the cross corresponds to $(\gamma, \Delta\chi_{1\sigma}^2) = (6/11, 1)$.

A. Constant growth index

Here we set γ_1 strictly equal to zero which implies $\gamma = \gamma_0$. If we fix the value of Ω_{m0} to that provided by WMAP7 ($\equiv 0.273$), then the corresponding vector \mathbf{p} contains only one free parameter namely, $\mathbf{p} = (0.273, \gamma)$. In figure 1 we plot the variation of $\Delta\chi^2 = \chi^2(\gamma) - \chi_{min}^2(\gamma)$ around the best γ fit value. Using the overall growth data (see solid line; $N = 11$) we find $\gamma = 0.616^{+0.088}_{-0.083}$ ($\chi_{min}^2/dof \simeq 6.33/10$), while for the *WiggleZ* data (dashed line; $N = 4$, entries 8-11) we obtain $\gamma = 0.64^{+0.099}_{-0.094}$ ($\chi_{min}^2/dof \simeq 1.72/3$), which is somewhat greater and almost 1σ ($\Delta\chi_{1\sigma}^2 = 1$) away, from the theoretically predicted value of $\gamma \simeq 6/11$ (see cross in figure 1). Such a discrepancy between the theoretical Λ CDM and observationally fitted value of γ has also been found by other authors. For example, Di Porto & Amendola [17] obtained $\gamma = 0.60^{+0.40}_{-0.30}$, Gong [18] measured $\gamma = 0.64^{+0.17}_{-0.15}$ while Nesseris & Perivolaropoulos [12], found $\gamma = 0.67^{+0.20}_{-0.17}$. Comparing the error bars among the various best fit values it is interesting to mention that including in the likelihood analysis the *WiggleZ* data we manage to reduce the error budget by $\sim 50\%$. In figure 2, we present the measured $f_{obs}(z)$ scaled to WMAP7 cosmology with the estimated growth rate function, $f(z) = \Omega_m^\gamma(z)$ [solid line]. Notice, that the open symbols corresponds to *WiggleZ* growth data.

B. Time varying growth index

Now we concentrate on the $\gamma(z)$ parametrizations, presented in section 3A, which means that $\mathbf{p} = (0.273, \gamma_0, \gamma_1)$. In figures 3 and 4 we present the results of our analysis for the Γ_1 , Γ_2 , Γ_3 and Γ_4 , models in the (γ_0, γ_1) plane. We sample $\gamma_0 \in [0.1, 1.3]$ and $\gamma_1 \in [-2.2, 2.2]$ in steps of 0.001. The contours are plot-

² Likelihoods are normalized to their maximum values. In the present analysis we always report 1σ uncertainties on the fitted parameters. Note that the uncertainty of the fitted parameters will be estimated, in the case of more than one such parameters, by marginalizing one with respect to the others.

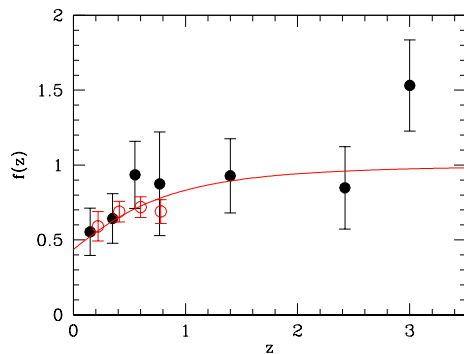


FIG. 2: Comparison of the observed, scaled to the WMAP7 cosmology (circles; see Table I and Appendix A), and theoretical evolution of the growth rate of clustering $f(z)$ [see solid line]. Note that the open circles corresponds to the *WiggleZ* growth data.

ted for 1σ , 2σ and 3σ confidence levels. In the left panels we show the contours using the old growth rate data (see Table I, entries 1-7; [12, 18, 47]) while in the right panel one can see results for the total sample including that of the *WiggleZ*. The theoretical (γ_0, γ_1) values in the Λ CDM model indicated by the crosses (see section 3A).

From the left panels of figures 3 and 4, it becomes clear that using the old growth rate data-set (entries 1-7) we are unable to place constraints on the (γ_0, γ_1) parameters. On the other hand, utilizing the overall growth rate sample, we find:

TABLE II: Statistical results for the overall data-set (see Table I): The 1st column indicates the $\gamma(z)$ parametrizations appearing in section 3A. 2nd and 3rd columns show the γ_0 and γ_1 best values. The remaining columns present the reduced χ^2 and the "reduction factor" S .

Model	γ_0	γ_1	χ^2_{min}/dof	S
Γ_1	$0.684^{+0.136}_{-0.124}$	$-0.331^{+0.507}_{-0.448}$	5.89/9	4.1
Γ_2	$0.453^{+0.135}_{-0.123}$	$0.566^{+0.449}_{-0.416}$	5.91/9	2.7
Γ_3	$0.424^{+0.135}_{-0.123}$	$0.739^{+0.247}_{-0.220}$	5.93/9	2.8
Γ_4	$0.90^{+0.136}_{-0.124}$	$-0.59^{+0.507}_{-0.448}$	5.84/9	2.1

(a) Γ_1 parametrization: In this case the likelihood function peaks at $\gamma_0 = 0.684^{+0.136}_{-0.124}$ and $\gamma_1 = -0.331^{+0.507}_{-0.448}$ with $\chi^2_{min}/dof \simeq 5.89/9$. Interestingly, the addition of four more points (*WiggleZ* growth data) in the statistical analysis provides a significant improvement in the derived (γ_0, γ_1) constraints. In particular, using the overall data-set we decrease the 2σ surface area (see left upper panel of fig.3) by a factor of ~ 4.1 . Below we will call this quantity "reduction factor" and is indicated by S , defined as the ratio of the surface area of the 2σ contour using the old growth data (entries: 1-7) to that of the total growth data-set. Actually, one would expect such an improvement because the *WiggleZ* survey measures $f(z)$

to within 9 – 17% in every redshift bin, in contrast to the old growth rate data [12, 18, 47] in which the corresponding accuracy lies in the interval 20 – 40%.

(b) Γ_2 and Γ_3 : Obviously, these parametrizations provide similar contours and thus they are almost equivalent as far as their statistics are concerned. We find that within 1σ errors we can put weak constraints on the free parameters. In particular, the best fit values are: (i) for Γ_2 we have $\gamma_0 = 0.453^{+0.135}_{-0.123}$, $\gamma_1 = 0.566^{+0.449}_{-0.416}$ and (ii) for Γ_3 model we obtain $\gamma_0 = 0.424^{+0.135}_{-0.123}$, $\gamma_1 = 0.739^{+0.247}_{-0.220}$. In both cases the reduced χ^2_{min}/dof is $\simeq 5.9/9$. Notice that the "reduction factor" here is $S \sim 2.7 - 2.8$.

(c) Γ_2 parametrization: In this case although the γ_0 is strongly degenerate with γ_1 , the likelihood function peaks at $\gamma_0 = 0.90^{+0.136}_{-0.124}$ and $\gamma_1 = -0.59^{+0.507}_{-0.448}$ with $\chi^2_{min}/dof \simeq 5.84/9$. Also we find that $S \sim 2.1$.

We would like to stress that the predicted (γ_0, γ_1) solutions (see section 3A) of the Γ_{1-4} parametrizations remain close to the 1σ borders (see crosses in figs.3,4). Finally, in Table II, one may see a more compact presentation of our results for the total sample, including the "reduction factor" due to the presence of the *WiggleZ* data.

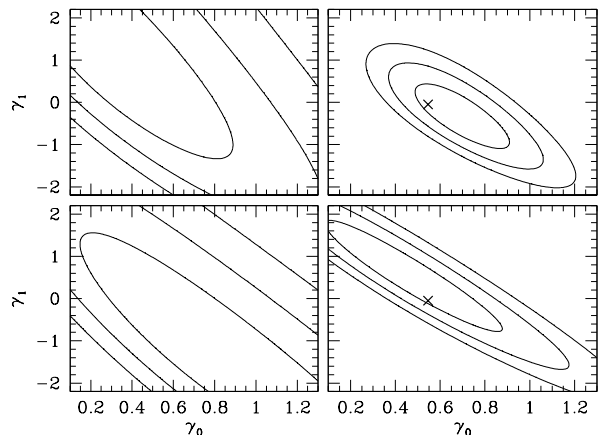


FIG. 3: Likelihood contours (for $\Delta\chi^2 = -2\ln\mathcal{L}/\mathcal{L}_{max}$ equal to 2.30, 6.16 and 11.81, corresponding to 1σ , 2σ and 3σ confidence levels) in the (γ_0, γ_1) plane in the case of Γ_1 (upper panel) and Γ_2 (bottom panel) parametrizations (see section 3A). In the left panels we present the contours that correspond to the old growth rate data (see Table I, entries 1-7) while the right panels show the likelihood contours for the overall sample including the *WiggleZ* data. We also include the theoretical Λ CDM ($\Omega_{m0} = 0.273$; crosses) pair $(\gamma_0, \gamma_1) = (6/11, -0.0478)$.

5. CONCLUSIONS

In this article we provide a general growth index evolution model $\gamma(z)$, based on phenomenology, which is

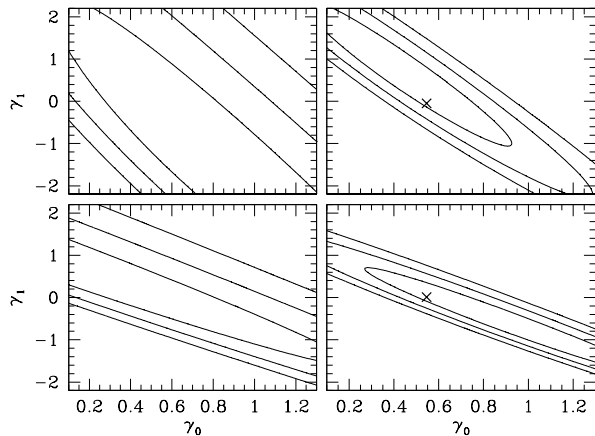


FIG. 4: The Likelihood contours for Γ_3 (upper panel) and Γ_4 (bottom panel). For more definitions see caption of figure 3. Here the crosses correspond to the theoretical (γ_0, γ_1) pair provided in section 3A [Γ_3 : $(6/11, -0.0478)$ and Γ_4 : $(6/11, 0.0113)$].

valid for all possible non-interacting dark energy models, including those of modified gravity. Armed with our general γ evolution model it is straightforward to apply the Polarski & Gannouji [27] approach to various $\gamma(z)$ models. In the context of the concordance Λ cosmology, we investigate the ability of five growth index parametrizations (including a constant one) to represent a variety of observational growth rate of structure data, based on 2dF, SDSS and *WiggleZ* measurements. To this end we perform a χ^2 minimization procedure between the observational growth data, after rescaling them to the WMAP7 cosmology, with the model expectations, through which we fit the model free parameters.

The comparison shows that all γ parametrizations fit at an acceptable level the current growth data, as indicated by the their reduced χ^2 values. Considering a constant growth index we can place tight constraints, up to $\sim 15\%$ accuracy, on the γ parameter. Indeed, for the total growth rate data-set (see Table I) we find that $\gamma = 0.616^{+0.088}_{-0.083}$, while using only the *WiggleZ* growth data we obtain $\gamma = 0.64^{+0.099}_{-0.094}$, which is 1σ away, from the theoretically predicted value of $\gamma \simeq 6/11$. Under the assumption that the growth index varies with time we find that the (γ_0, γ_1) parameter solution space of all

growth index parametrizations, accommodate the theoretical (γ_0, γ_1) values at 2σ level. We also observe that the inclusion of the new *WiggleZ* data reduce significantly the (γ_0, γ_1) parameter solution space. Despite the latter improvement we find that the majority of the $\gamma(z)$ parametrizations still suffer from the $\gamma_0 - \gamma_1$ degeneracy, implying that more and accurate data are essential.

Appendix A: Scaling the growth rate data to the same cosmology

The observed growth rate of clustering is given by $f_{obs} = \beta b$, where $\beta(z)$ is the redshift space distortion parameter and $b(z)$ is the linear bias. Observationally, using the anisotropy of the correlation function one can estimate the $\beta(z)$ parameter. The linear bias factor can be defined as the ratio of the variances of the tracer (galaxies, QSOs etc) and underlying mass density fields, smoothed at some linear scale traditionally taken to be $8h^{-1}$ Mpc (at which scale the variance is of order unity): $b(z) = \sigma_{8,tr}(z)/\sigma_8(z)$. Now, since different authors have estimated the observed growth rate of structure using different cosmologies (see our Table I), we need to convert them to the same cosmological background in order to be able to use them consistently. Note that as a background cosmology we choose $(\Omega_{m0}, \sigma_8) = (0.273, 0.811)$. In particular, we wish to translate the value of growth rate from one cosmological model, say *Ref* (see Table I), to the background cosmology. The definition of $f(z)$ and $b(z)$ simply implies:

$$\frac{f_{obs}(z)}{f_{obs,Ref}(z)} = \frac{\beta(z)}{\beta_{Ref}(z)} \frac{\sigma_{8,tr}(z)}{\sigma_{8,tr,Ref}(z)} \frac{\sigma_{8,Ref}(z)}{\sigma_8(z)}, \quad (A.1)$$

where $\sigma_8(z) = \sigma_8 D(z)$ and $\sigma_{8,Ref}(z) = \sigma_{8,Ref} D_{Ref}(z)$. Note that the growth factor is given by Eq.(3.3). Making the fair assumption that:

$$\sigma_{8,tr}(z) \simeq \sigma_{8,tr,Ref}(z) \quad \beta(z) \simeq \beta_{Ref}(z), \quad (A.2)$$

since the different cosmologies enter only weakly in the observational determination of $\sigma_{8,tr}(z)$ and $\beta(z)$, through the definition of distances, we then have:

$$f_{obs}(z) \simeq f_{obs,Ref}(z) \frac{\sigma_{8,Ref}}{\sigma_8} \frac{D_{Ref}(z)}{D(z)}. \quad (A.3)$$

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